

# The ARCH Model

## Full Name:

The **A**utoregressive **C**onditional **H**eteroskedasticity Model

## Mathematical Notation:

$$\text{Var}(y_t | y_{t-1}) = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

$\text{Var}(y_t | y_{t-1})$  → Conditional variance

$\alpha_0$  → Constant factor  $\approx c$

$\alpha_1$  → Coefficient associated with the first term  $\approx \theta_1$

$\epsilon_{t-1}^2$  → Squared value of the residual epsilon for the previous period

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## Description:

Unlike the previous models, the ARCH measures volatility of the results, rather than the results themselves. Thus, the purpose of it is entirely different and focused on predicting turbulence in the data, regardless of whether it's an increase or decrease in the values.

As you can see on the left side of the equation, the endogenous variable is the variance, rather than the time series variable.

Thus, this is only the variance equation of the model. The simplest ARCH model assumes a 0 or constant mean, so this is the only equation we are interested in.

# The ARCH Model

## Implementation of the Simple Model in Python:

The library the *arch\_model* method comes from

```
from arch import arch_model
```

The method we are importing

```
model_arch_1 = arch_model(df.returns[1:], mean = "Constant", vol = "ARCH", p = 1)
```

The type of volatility model we are assuming for the volatility equation

The variable storing the model characteristics that we will fit later

The time series, whose volatility we wish to analyse

The type of model we are assuming for the mean equation

The order of the model  
\*For an ARCH(q) model, simply change p from 1 to q.